3 (Sem–3/CBCS) MAT HG 1/RC/HG 2

2021

(Held in 2022)

MATHEMATICS

(Honours Generic/Regular)

Paper : MAT–HG–3016 /MAT–RC–3016

(Differential Equations)

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

*Answer **either** in English **or** in Assamese.*

OPTION–A

1. Answer the following questions : 1×10=10

তলত দিয়া প্ৰশ্নবোৰৰ উত্তৰ কৰা ঃ

(*a*) Write down the order of the following differential equation :

তলৰ অৱকল সমীকৰণটোৰ ক্ৰম লিখা ঃ

$$\left(\frac{dr}{ds}\right)^3 = \sqrt{\frac{d^2r}{ds^2}+1}$$

Contd.

(b) State whether the following differential equation is linear or nonlinear :

তলৰ অৱকল সমীকৰণটো ৰৈখিক নে অৰৈখিক লিখা ঃ

$$\frac{d^2y}{dx^2}+5\frac{dy}{dx}+6y^2=0$$

(c) Form the differential equation of the family of parabolas $y=cx^2$.

$y=cx^2$ অধিবৃত্ত পৰিয়ালটোৰ অৱকল সমীকৰণটো গঠন কৰা।

(d) Write down the condition under which the n solutions $f_1, f_2, \ldots, f_n$ of an nth order homogeneous linear differential equation are linearly independent on $a \leq x \leq b$.

এটা n ক্ৰমৰ সমমাত্ৰিক ৰৈখিক অৱকল সমীকৰণৰ n টা সমাধান $f_1, f_2, \ldots, f_n$ য়ে $a \leq x \leq b$ অন্তৰালত ৰৈখিকভাৱে স্বতন্ত্ৰ হোৱাৰ চৰ্ত্তটো লিখা।

(e) Determine the integrating factor of the following linear differential equation :

$$x^4 \frac{dy}{dx} + 2x^3 y = 1$$

তলৰ বৈখিক অৱকল সমীকৰণটোৰ অনুকলন গুণক উলিওৱা ঃ

$$x^4 \frac{dy}{dx} + 2x^3 y = 1$$

(f) What is meant by integral curves of a differential equation ?

এটা অৱকল সমীকৰণৰ সমাকল লেখ (Integral curves) বুলিলে কি বুজা ?

(g) Write *one* special characteristic of Cauchy-Euler equation.

ক'চি-ইউলাৰ সমীকৰণৰ *এটা* বিশেষ বৈশিষ্ট্য লিখা।

(h) Evaluate the Wronskian of the functions

$$f_1(x) = e^x,\ f_2(x) = e^{-x}$$

$f_1(x) = e^x,\ f_2(x) = e^{-x}$ ফলন দুটাৰ Wronskian নিৰ্ণয় কৰা।

(i) Write down the UC set corresponding to the UC function x^n.

UC ফলন x^n সাপেক্ষে UC সংহতিটো লিখা।

(j) Determine the constant A in

$$\left(x^2+3xy\right)dx+\left(Ax^2+4y\right)dy=0$$

such that the equation is exact.

$$\left(x^2+3xy\right)dx+\left(Ax^2+4y\right)dy=0$$

সমীকৰণটো যথাৰ্থ হ'লে, ধ্ৰুৱক A ৰ মান নিৰ্ণয় কৰা।

2. Answer the following questions : 2×5=10

তলত দিয়া প্ৰশ্নবোৰৰ উত্তৰ কৰা ঃ

(a) Show that $f(x)=2\sin x+3\cos x$ is a solution of the differential equation $\frac{d^2y}{dx^2}+y=0$.

State whether it is an implicit or explicit solution.

দেখুওৱা যে, $\frac{d^2y}{dx^2}+y=0$ অৱকল সমীকৰণটোৰ $f(x)=2\sin x+3\cos x$ এটা সমাধান হয়। এই সমাধানটো অন্তর্নিহিত নে শুপ্রকাশিত (explicit) উল্লেখ কৰা।

(b) Determine the most general function $N(x,y)$ such that the equation $(x^3+xy^2)dx+N(x,y)dy=0$ is exact.

অত্যন্ত সাধাৰণ ফলন $N(x,y)$ উলিওৱা যাতে, $(x^3+xy^2)dx+N(x,y)dy=0$ সমীকৰণটো যথাৰ্থ হয়।

(c) Find the general solution of —

সাধাৰণ সমাধান উলিওৱা —

$$\frac{d^2y}{dx^2}-2\frac{dy}{dx}-3y=0$$

(d) Solve :

সমাধান কৰা ঃ $4xy\,dx+\left(x^2+1\right)dy=0$

(e) Reduce the Bernoulli's equation $\frac{dy}{dx}+y=xy^3$ to linear equation by appropriate transformation.

উপযুক্ত ৰূপান্তৰৰ সহায়ত বাৰ্নোলীৰ সমীকৰণ $\frac{dy}{dx}+y=xy^3$ক ৰৈখিক সমীকৰণলৈ সমানীত কৰা।

3. Answer ***any four*** of the following questions : 5×4=20

তলত দিয়াবোৰৰ *যিকোনো চাৰিটা* প্ৰশ্নৰ উত্তৰ কৰা ঃ

(a) Show that $x^3+3xy^2=1$ is an implicit solution of the differential equation $2xy\frac{dy}{dx}+x^2+y^2=0$ on the interval $0<x<1$.

দেখুওৱা যে, $0<x<1$ অন্তৰালত $2xy\frac{dy}{dx}+x^2+y^2=0$ অৱকল সমীকৰণটোৰ $x^3+3xy^2=1$ এটা অন্তর্নিৰ্হিত সমাধান হয়।

(b) If $M(x,y)dx+N(x,y)dy=0$ is a homogeneous equation, then the change of variables $y=vx$ transforms it into a separable equation in the variables v and x— Prove it.

প্ৰমাণ কৰা যে, $M(x,y)dx+N(x,y)dy=0$ এটা সমমাত্ৰিক সমীকৰণ হ'লে $y=vx$ চলক সলনীকৰণেৰে ইয়াক v আৰু x চলকৰ পৃথকীকৰণ সমীকৰণত প্ৰকাশ কৰিব পাৰি।

(c) Solve the following initial value problem :

তলৰ আদি মান যুক্ত সমীকৰণটো সমাধান কৰা ঃ

$$\frac{dy}{dx}+\frac{y}{2x}=\frac{x}{y^3},\ y(1)=2$$

(d) Find the general solution of $\frac{d^2y}{dx^2}+6\frac{dy}{dx}+5y=2e^x+10e^{5x}$ by the method of undermined co-efficients.

অনিৰ্ধাৰিত সহগ পদ্ধতিৰে

$$\frac{d^2y}{dx^2}+6\frac{dy}{dx}+5y=2e^x+10e^{5x}$$

সমীকৰণটোৰ সাধাৰণ সমাধান উলিওৱা।

(e) Solve (সমাধান কৰা) ঃ

$$(x+2y+3)\,dx+(2x+4y-1)\,dy=0$$

(f) Solve the initial value problem :

আদিমান যুক্ত সমীকৰণটো সমাধান কৰা ঃ

$$\frac{d^2y}{dx^2}-4\frac{dy}{dx}+13y=0$$

$$y(0)=2,\ y'(0)=7$$

4. Answer ***any four*** of the following questions : 10×4=40

তলৰ *যিকোনো চাৰিটা* প্ৰশ্নৰ উত্তৰ কৰা ঃ

(a) Consider the following differential equation :

$$(4x+3y^2)\,dx+2xy\,dy=0$$

$$(4x+3y^2)\,dx+2xy\,dy=0$$

অৱকল সমীকৰণটোৰ ক্ষেত্ৰত

(i) Show that the equation is not exact ;

দেখুওৱা যে, সমীকৰণটো যথাৰ্থ নহয় ;

(ii) Find an integrating factor of the form x^n, where n is a positive integer.

এটা অনুকলন গুণক x^n উলিওঁৱা, য'ত n এটা ধনাত্মক অখণ্ড সংখ্যা হয় ;

(iii) Multiply the equation by the integrating factor and solve the resulting exact equation.

সমীকৰণটো অনুকলন গুণকেৰে পূৰণ কৰা আৰু লব্ধ যথাৰ্থ সমীকৰণটো সমাধান কৰা।

1+3+6=10

(b) Find the general solution of

সাধাৰণ সমাধান উলিওৱা ঃ

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 2e^x - 10\sin x$$

(c) (i) Find the orthogonal trajectories of the family of circles which are tangent to the y-axis at the origin. 5

মূলবিন্দুত y অক্ষক স্পৰ্শ কৰি থকা বৃত্তৰ পৰিয়ালটোৰ লাম্বিক প্ৰক্ষেপ পথ (orthogonal trajectory) নিৰ্ণয় কৰা।

(ii) Find a family of oblique trajectories that intersect the family of parabolas $y^2 = cx$ at an angle $60°$. 5

$y^2 = cx$ অধিবৃত্তৰ পৰিয়ালটোক $60°$ কোণত ছেদ কৰি থকা এটি তিৰ্যক প্ৰক্ষেপ পথ (oblique trajectory)ৰ পৰিয়াল উলিওৱা।

(d) Solve by the method of variation of parameter :

প্ৰাচল বিচৰণ পদ্ধতিৰে সমাধান কৰা ঃ

$$\frac{d^2y}{dx^2} + y = \sec x$$

(e) (i) Given that $y = x$ is a solution of

$$\left(x^2+1\right)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$$

Find a linearly independent solution by reducing the order. 6

$\left(x^2+1\right)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$ অৱকল

সমীকৰণটোৰ $y = x$ এটা সমাধান হয়। সমীকৰণটোৰ ক্ৰম লঘুকৃত (সমানীত) কৰি এটা বৈখিকভাৱে স্বতন্ত্ৰ সমাধান উলিওৱা।

(ii) Show that x and x^2 are linearly independent solution of equation

$$x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$$

Also find the solution that satisfies the conditions $y(1) = 3$, $y'(1) = 2$.

2+2=4

দেখুওৱা যে, $x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$ সমীকৰণটোৰ x আৰু x^2 দুটা ৰৈখিকভাৱে স্বতন্ত্ৰ সমাধান।

লগতে $y(1) = 3$, $y'(1) = 2$ চৰ্ত সাপেক্ষে ইয়াৰ সমাধান উলিওৱা।

(f) Solve (সমাধান কৰা) :

$$x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = 4\ln x$$

(g) Consider the linear system

ৰৈখিক সমীকৰণ প্ৰণালী এটা লোৱা হ'ল

$$\frac{dx}{dt} = 3x + 4y$$

$$\frac{dy}{dt} = 2x + y$$

(i) Show that (দেখুওৱা যে)

$x=2e^{5t}$, $x=e^{-t}$

and (আৰু)

$y=e^{5t}$, $y=-e^{-t}$

are solutions of this system

(এই প্ৰণালীটোৰ সমাধান হয়)।

(ii) Show that the two solutions of part *(i)* are linearly independent on every interval $a \le t \le b$.

দেখুওৱা যে part *(i)* ত উল্লিখিত সমাধান দুটা $a \le t \le b$ অন্তৰালত ৰৈখিকভাৱে স্বতন্ত্ৰ হয়।

(iii) Write the general solution of the system.

Also find the solution

$x=f(t)$, $y=g(t)$

for which $f(0)=1$ and $g(0)=2$.

প্ৰণালীটোৰ সাধাৰণ সমাধান লিখা। লগতে $f(0)=1$ আৰু $g(0)=2$ চৰ্ত সাপেক্ষে প্ৰণালীটোৰ সমাধান $x=f(t)$, $y=g(t)$ উলিওঁৱা। 5+2+3=10

(h) Solve the following : 5+5=10

তলত দিয়াবোৰৰ সমাধান উলিওৱা ঃ

(i) $\frac{dy}{dx}+y=f(x)$ where (য'ত)

$$f(x)=\begin{cases}2, & 0 \le x < 1 \\ 0, & x \ge 1\end{cases}, \; y(0)=0$$

(ii) $\frac{d^2y}{dx^2}-y=3x^2e^x$

Paper : MAT–HG–3026

(Linear Programming)

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

OPTION–B

1. Choose the correct option : 1×10=10

   (i) The linear programming problem (LPP)

   Maximize $x_1 + x_2$

   subject to $x_1 + x_2 \le 1$

   $-3x_1 + x_2 \ge 3$

   $x_1, x_2 \ge 0$

   has

   (a) no feasible solution

   (b) unique optimal solution

   (c) alternate optimal solution

   (d) unbounded solution

(ii) A basic feasible solution (B.F.S) to an LPP is called degenerate, if

(a) all the basic variables are zero

(b) at least one of the basic variables is zero

(c) at most one of the basic variables is zero

(d) none of the basic variables is zero

(iii) Which of the following statement(s) is/ are correct ?

Statement I : A B.F.S. to an LPP must correspond to an extreme point of the covex set of all the feasible solutions to the LPP.

Statement II : Every extreme point of the convex set of all the feasible solutions to an LPP is a B.F.S.

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

(iv) The optimal value of the objective function of the LPP

Maximum $3x_1 + 2x_2$

subject to $x_1 + x_2 \le 6$

$$2x_1 + x_2 \le 6$$

$$x_1, x_2 \ge 0$$

is obtained at the point

(a) $(2, 3)$

(b) $(3, 2)$

(c) $(0, 6)$

(d) $(6, 0)$

(v) If an LPP has a feasible solution, then

(a) it also has a B.F.S

(b) it has infinite number of B.F.S.

(c) it can never have a B.F.S.

(d) it cannot have an optimal solution

(vi) Choose the incorrect statement :

(a) The convex combination of a finite number of optimal solutions to an LPP is again an optimal solution to the problem.

(b) For the solution of any LPP by simplex method, the existence of initial B.F.S. is always assumed.

(c) Big-M method is used to find the solution of LPP having artificial variables.

(d) In phase I of the two-phase simplex method, the sum of the artificial variables is maximized subject to the given constraints.

(vii) Choose the incorrect statement :

(a) The dual of the dual is the primal.

(b) In a primal-dual pair, the dual problem must always be of the minimization type.

(c) The optimal values of the primal objective function and that of its dual are same.

(d) If the primal problem has m constraints in n variables, then its dual will have n constraints in m variables.

(viii) A transportation problem is balanced, if

(a) the number of sources equals the number of destinations

(b) there is no real distinction between sources and destinations

(c) total demand equals total supply irrespective of the number of sources and destinations

(d) total demand and total supply are equal and the number of sources equals the number of destinations

(ix) In an assignment problem involving six workers and five jobs, total number of assignments possible is

(a) 5

(b) 6

(c) 11

(d) 30

(x) If the value of a game is zero, then it is called

(a) finite game

(b) infinite game

(c) fair game

(d) unfair game

2. Answer the following questions : 2×5=10

(a) Solve the following LPP graphically :

Maximize $2x_1 + 3x_2$

subject to $x_1 + 2x_2 \leq 4$

$x_1 + x_2 \leq 3$

$x_1, x_2 \geq 0$

(b) Show that the intersection of two convex sets is also a convex set.

(c) Examine whether the following LPP has a degenerate B.F.S. :

Maximize $4x_1 + 5x_2 + x_3$

subject to $2x_1 + x_2 - x_3 = 2$

$3x_1 + 2x_2 + x_3 = 3$

$x_1, x_2, x_3 \geq 0$

(d) Write down the dual of the following LPP :

Minimize $4x_1 + 6x_2 + 18x_3$

subject to $x_1 + 3x_2 \geq 3$

$x_1 + 2x_3 \geq 5$

$x_1, x_2, x_3 \geq 0$

(e) Use North-West Corner method to find an initial basic feasible solution to the following transportation problem :

	1	2	3	4	supply
1	3	7	6	4	5
2	2	4	3	2	2
3	4	3	8	5	3
Demand	3	3	2	2	

3. Answer ***any four*** of the following : 5×4=20

(a) Show that the set of feasible solutions to an LPP is a convex set.

(b) Obtain all the basic solutions to the LPP —

Maximize $x_1 + 3x_2 + x_3$

subject to $x_1 + 2x_2 + x_3 = 4$

$2x_1 + x_2 + 5x_3 = 5$

$x_1, x_2, x_3 \geq 0$

(c) Show that the following LPP has unbounded solution :

Maximize $2x_1 + x_2$

subject to $x_1 - x_2 \leq 10$

$2x_1 - x_2 \leq 40$

$x_1, x_2 \geq 0$

(d) Solve the dual of the following LPP :

Maximize $3x_1 - 2x_2$

subject to $x_1 \leq 4$

$x_2 \leq 6$

$x_1 + x_2 \leq 5$

$x_2 \geq 1$

$x_1, x_2 \geq 0$

(e) Use Vogel's Approximation method to obtain an initial B.F.S. to the following transportation problem :

	1	2	3	Supply
1	16	20	12	200
2	14	8	18	160
3	26	24	16	90
Demand	180	120	150	

(f) The pay-off matrix of a two-person game is given below :

		B		
		I	II	III
	I	1	3	1
A	II	0	− 4	− 3
	III	1	5	−1

Find the best strategy of each player and the value of the game.

4. (a) If $x_1=2$, $x_2=4$ and $x_3=1$ is a feasible solution to the LPP

Maximum $5x_1 - 6x_2 + 7x_3$

subject to $2x_1 - x_2 + 2x_3 = 2$

$x_1 + 4x_2 = 18$

$x_1, x_2, x_3 \geq 0$,

reduce it to a basic feasible solution. 10

Or

Use simplex method to solve the LPP —

Maximum $x_1 - 3x_2 + 2x_3$

subject to $3x_1 - x_2 + 3x_3 \le 7$

$$-2x_1 + 4x_2 \le 12$$

$$-4x_1 + 3x_2 + 8x_3 \le 10$$

$$x_1, x_2, x_3 \ge 0$$

(b) Use two-phase simplex method to solve the LPP — 10

Minimize $x_1 + x_2$

subject to $2x_1 + x_2 \ge 4$

$$x_1 + 7x_2 \ge 7$$

$$x_1, x_2 \ge 0$$

Or

Use Big-M method to solve the LPP—

Maximize $3x_1 - x_2$

subject to $2x_1 + x_2 \ge 2$

$$x_1 + 3x_2 \le 3$$

$$x_2 \le 4$$

$$x_1, x_2 \ge 0$$

(c) Write down the solution to the following LPP by solving its dual : 10

Minimize $15x_1 + 10x_2$

subject to $3x_1 + 5x_2 \geq 5$

$5x_1 + 2x_2 \geq 3$

$x_1, x_2 \geq 0$

Or

State and prove the complementary slackness theorem.

(d) Find an optimal solution to the following transportation problem : 10

	1	2	3	4	supply
1	3	6	8	5	20
2	6	1	2	5	28
3	7	8	3	9	17
Demand	15	19	13	18	

Or

Apply the Hungarian method to solve the following assignment problem :

	I	II	III	IV
A	87	85	71	38
B	91	89	75	34
C	70	72	86	75
D	37	35	21	88