Total number of printed pages–7

3 (Sem–1 /CBCS) PHY HC 1

2021

(Held in 2022)

PHYSICS

(Honours)

Paper : PHY–HC–1016

(Mathematical Physics –I)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : $1\times7=7$

   (a) State the vector field with respect to Cartesian co-ordinate. Give *one* example.

   (b) Show that $\vec{\nabla}.\vec{r}=3$, where $\vec{r}=\hat{i}x+\hat{j}y+\hat{k}z$.

Contd.

(c) Write the order and degree of the differential equation

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$

(d) Write the volume element in curvilinear co-ordinate.

(e) Give the value of $\int_{-\alpha}^{+\alpha} \delta(x)\, dx$

(f) Define variance in statistics.

(g) State the principle of least square fit.

2. Answer of the following questions : 2×4=8

(a) Find a unit vector perpendicular to the surface, $x^2 + y^2 - z^2 = 11$ at the point $(4,2,3)$.

(b) If $\vec{A} = \vec{A}(t)$, then show that

$$\frac{d}{dt}\left[\vec{A}.\left(\frac{d\vec{A}}{dt} \times \frac{d^2\vec{A}}{dt^2}\right)\right] = A.\left[\frac{d\vec{A}}{dt} \times \frac{d^3\vec{A}}{dt^3}\right]$$

(c) If $\vec{A}$ and $\vec{B}$ are each irrotational, prove that $\vec{A} \times \vec{B}$ is solenoidal.

(d) Evaluate $\iint_S \vec{r} \times \hat{n} dS$, where S is a closed surface.

3. Answer ***any three*** of the following questions :

5×3=15

(a) Prove

$$\iiint_V \left(\phi \nabla^2 \psi - \psi \nabla^2 \phi\right) dV = \iint_S \left(\phi \vec{\nabla} \psi - \psi \vec{\nabla} \phi\right) . dS$$

(b) Find the integrating factor (IF) of the following differential equation and solve it.

$$\left(1 + x^2\right) \frac{dy}{dx} + 2xy = \cos x$$

(c) Express curl $\vec{A} = \vec{\nabla} \times \vec{A}$ in cylindrical co-ordinate.

(d) What is Dirac-delta function ? Show that the function

$$\delta(x) = \lim_{\varepsilon \to 0} \frac{\sin(2\pi\varepsilon x)}{\pi\varepsilon}$$

is a Dirac delta function.

(e) If $\phi(x,y,z) = 3x^2y - y^3x^2$ be any scalar function ϕ, find out

(i) grad ϕ at point (1, 2, 2)

(ii) unit vector $\hat{e}$ perpendicular to surface.

4. Answer ***any three*** of the following questions : 10×3=30

(a) (i) If $F_1(x,y)$, $F_2(x,y)$ are two continuous functions having continuous partial derivatives $\frac{\partial F_1}{\partial y}$ and $\frac{\partial F_2}{\partial x}$ over a region R bounded by simple closed curve C in the x–y plane, then show that

$$\oint_C (F_1 dx + F_2 dy) = \iint_S \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right) dx\,dy$$

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(ii) A function $f(x)$ is defined

as $$\begin{cases} 0, & x<2 \\ \frac{1}{18}(2x+3), & 2 \le x \le 4 \\ 0, & x>2 \end{cases}$$

Show that it is a probability density function. 3

(b) Solve the following differential equations : 5+5=10

(i) $$9\frac{d^2y}{dx^2}+12\frac{dy}{dx}+4y=6e^{-2x/3}$$

(ii) $$2\frac{\partial^2 z}{\partial x^2}+5\frac{\partial^2 z}{\partial x \partial y}+2\frac{\partial^2 z}{\partial y^2}=0$$

(c) (i) A rigid body rotates about an axis passing through the origin with angular velocity $\vec{\omega}$ and with linear velocity $\vec{v}=\vec{\omega}\times\vec{r}$, then prove that,

$$\vec{\omega}=\frac{1}{2}\left(\vec{\nabla}\times\vec{v}\right)$$

where, $\vec{\omega}=\hat{i}\omega_1+\hat{j}\omega_2+\hat{k}\omega_3$

$$\vec{r}=\hat{i}x+\hat{j}y+\hat{k}z$$ 5

(ii) If $y = f(x+at) + g(x-at)$, show that it satisfies the equation

$$\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$$

where f and g are assumed to be at least twice differentiable and a is any constant. 5

(d) (i) Apply Green's theorem in plane to evaluate the integral

$\oint_C \left[\left(xy - x^2\right) dx + x^2 y\, dy\right]$ over the triangle bounded by the line $y=0$, $x=1$ and $y=x$. 6

(ii) Prove that

$$\int_{-\alpha}^{+\alpha} f(x)\, \delta(x-c)\, dx = f(c)$$ 4

(e) (i) Applying Gauss' theorem, evaluate

$$\iint_S x\,dydz + y\,dzdx + z\,dxdy,$$ where

S is the sphere of radius

$x^2 + y^2 + z^2 = 1$ 5

(ii) Evaluate $\nabla^2\psi$ in spherical co-ordinate. 5
