Total number of printed pages–8

3 (Sem–5/CBCS) PHY HC 1

2022

PHYSICS

(Honours)

Paper : PHY–HC–5016

(Quantum Mechanics and Applications)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer ***any seven*** of the following : $1\times7=7$

(a) Write down the expression of wave function of matter wave associated with a free particle travelling along the *x*-axis having momentum *p* and energy *E*.

(b) Is the function $\psi = a\cos m\phi$ an eigen-function of *z*-component of angular momentum operator ? Give reason.

(c) Why $\psi = ax^2$ is not an acceptable wave function in quantum mechanics ?

Contd.

(d) Write down the condition of orthogonality of wave function.

(e) When does the quantum mechanical probability of oscillator become identical with the classical probability?

(f) How do you represent dynamical variables in quantum mechanics ?

(g) The number of permitted eigenvalues in a finite potential well is :

(i) two

(ii) zero

(iii) fixed for well of any height

(iv) variable, depending on the height of the well

(h) What is the total number of energy level (or degeneracy) for the nth state of hydrogen atom ?

(i) Why does the normal Zeeman effect occur only in atoms with even number of electrons ?

(j) What is the need of an inhomogeneous magnetic field in Stern-Gerlach experiment ?

(k) The spin-orbit interaction has no effect on

(i) f-state

(ii) δ-state

(iii) d-state

(iv) p-state

(l) What is Bohr magneton ?

2. Answer ***any four*** of the following : 2×4=8

(a) What are the conditions and limitations that the wave function must obey ?

(b) Show that $\left[\hat{x}^n, \hat{P}_x\right] = i\hbar n x^{n-1}$

(c) The one-dimensional wave function is given by $\psi(x) = \sqrt{a}\, e^{-ax}$. Find the probability of finding the particle between $x = \frac{1}{a}$ and $x = \frac{2}{a}$.

(d) What is a Gaussian wave packet ? Express its wave function.

(e) For a free particle, show that the group velocity of the wave packet is equal to the classical velocity of the particle.

(f) What do you mean by Larmor precession ? What is Larmor frequency ?

(g) Determine the value of spin magnetic momentum of free electron.

(h) Determine the possible values of resultant angular momentum for two electrons having $j_1 = \frac{3}{2}$ and $j_2 = -\frac{5}{2}$.

3. Answer ***any three*** of the following : 5×3=15

(a) In what respect does Schrödinger equation differ from classical wave equation ? Obtain the three-dimensional time independent Schrödinger equation from the time-dependent form. 1+4=5

(b) What do you understand by the wave function ψ of a moving particle ? Give its physical significance. What does the square of wave function signify ? 1+2+2=5

(c) What is an operator ? Write the expression for position operator, momentum operator and energy operator. What is Hamiltonian operator ? 1+3+1=5

(d) Use the time independent Schrödinger equation to find $V(x)$ and energy E for which the wave function is

$$\psi(x) = \left(\frac{x}{x_0}\right)^n e^{-x/x_0}$$

where n and x_0 are constants.

Assume $V(x) \to 0$, as $x \to \infty$.

(e) What is the significance of zero point energy ? Calculate the zero point energy for an oscillating mass of $1gm$ connected to a spring, which is stretched $1cm$ by a force $0{\cdot}1N$. The particle is constrained to move along the x-axis. 2+3=5

(f) Derive an expression for orbital magnetic moment due to electron rotating arround the nucleus of an atom. What is Bohr magneton ? 4+1=5

(g) What is Lande's g-factor ? Calculate the Lande's g-factor for the $3P_1$ state. Use the result to predict the splitting of the energy level when the atom is placed in an external magnetic field of 0.1 Tesla.
(Given Bohr magneton = $9.3 \times 10^{-24} Am^2$)
1+1+3=5

(h) The atomic number of Beryllium is 4. Determine its

(i) electronic configuration in ground state.

(ii) electronic configuration in first excited state.

(iii) spectroscopic terms in the ground state.

(iv) spectroscopic terms in the first excited state.

4. Answer ***any three*** of the following : 10×3=30

(a) (i) What is the need for differential wave equation ? Starting from the wave equation and introducing energy and momentum of the particle, obtain expression for one dimensional Schrödinger time dependent wave equation for waves associated with a moving particle in a potential field *V*. 1+4=5

(ii) What is the physical significance of $\int_{-\infty}^{+\infty} |\psi|^2 \, dx = 1$?

A particle is represented by the wave function $\psi(x) = Ae^{-|x|} \sin \alpha x$ Evaluate the normalization constant *A*, α being a constant. 1+4=5

(b) (i) Calculate the normalization constant for a wave function (at $t = 0$) given by $\psi(x) = ae^{-(\alpha^2 x^2)/2} e^{ikx}$ known as Gaussian wave packet. Determine.

(ii) the probability density and

(iii) probability current density for this function.

4+2+4=10

(c) Write the radial equation of Hydrogen atom and solve it for obtaining its energy eigenvalues. 2+8=10

(d) Discuss classical and quantum interpretations of square well potential of finite depth and determine allowed values of energy using graphical method. Also sketch the wave functions corresponding to three lowest energy levels.

(e) (i) What are symmetric and anti-symmetric wave functions ?

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(ii) State and explain Pauli's exclusion principle. How does a knowledge of symmetric and anti-symmetric wave functions lead to this principle ? 5+3=8

(f) Describe and explain *LS* and *JJ* couplings. Illustrate them with vector diagram. Give the selection rules for *L*, *S* and *J*. 2+2+4+2=10

(g) Elucidate anomalous Zeeman effect using concept of 'spin' of electrons. Illustrate the anomalous Zeeman effect for Sodium D-lines. 6+4=10

(h) Write short notes on *any two* of the following : 5×2=10

(i) Stern-Gerlach experiment

(ii) Paschen-Back effect

(iii) Stark effect

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